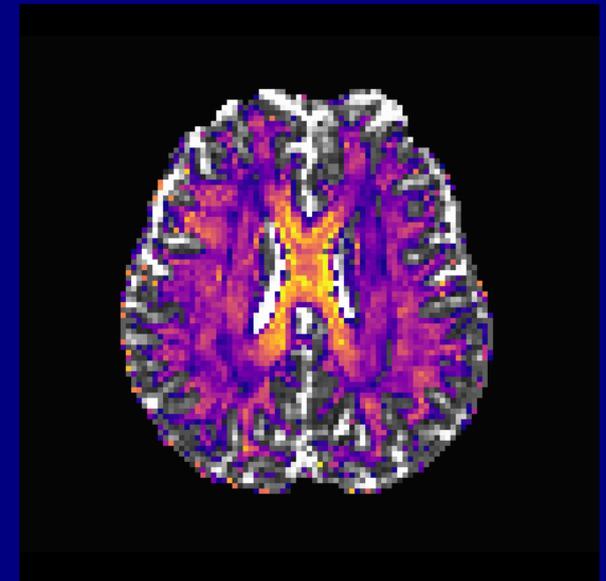
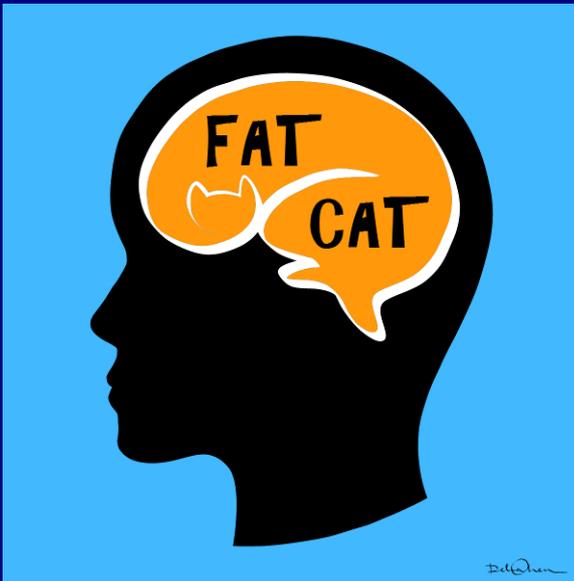


# Introduction to: DWI + DTI

**AFNI Bootcamp (SSCC, NIMH, NIH)**



# Outline

- + DWI and DTI
  - Concepts behind diffusion imaging
  - Diffusion imaging basics in brief
  - Connecting DTI parameters and geometry
  - Role of noise+distortion →DTI parameter uncertainty

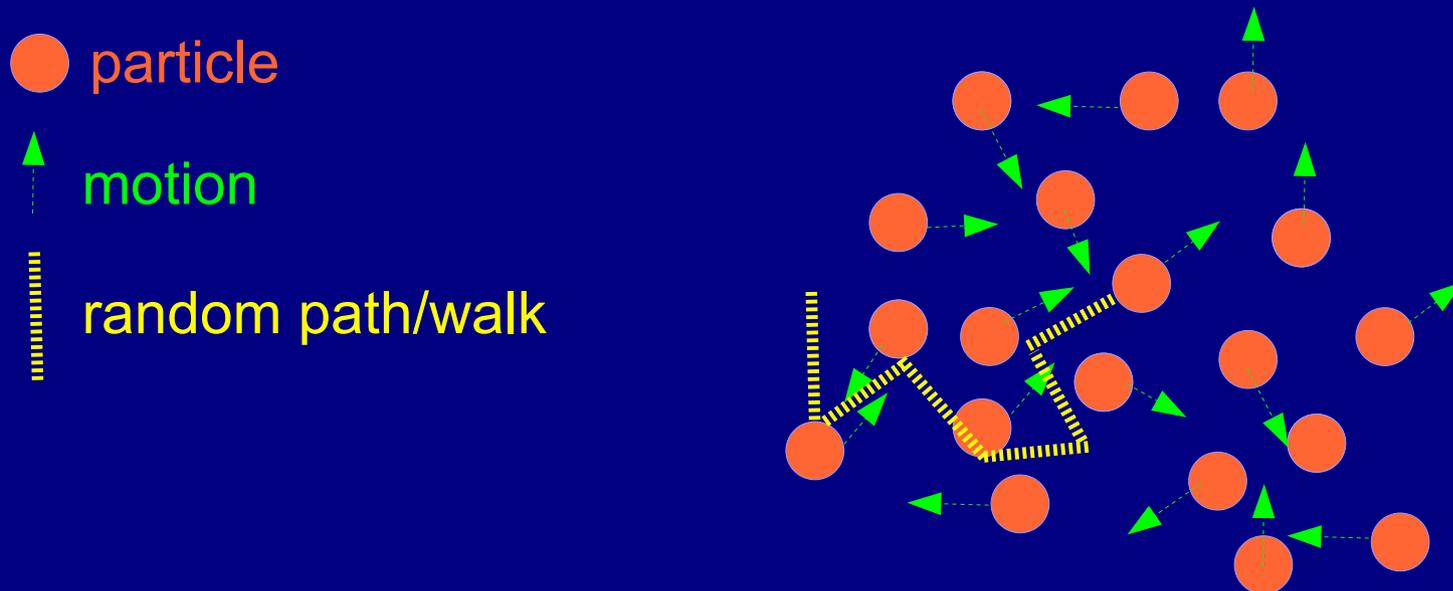
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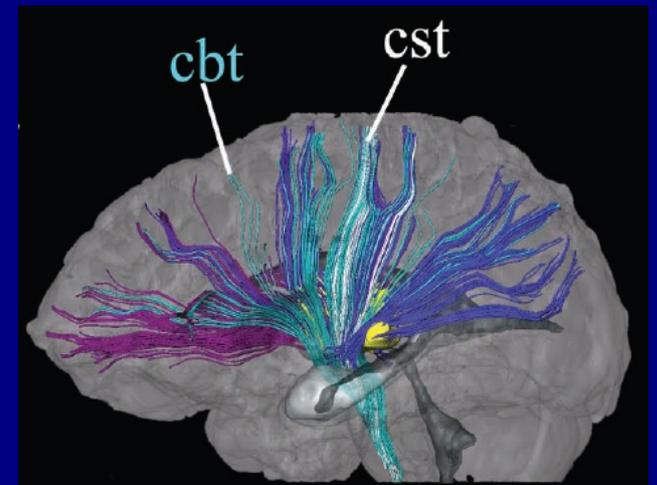
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**Imaging:** quantifying brain properties  
→ here, esp. for white matter



*The DTI model:*

Assumptions and relation to WM properties

# Diffusion as environmental marker

Diffusion: random (Brownian) motion of particles → mixing or spreading

Ex: unstirred, steeping tea (in a large cup):



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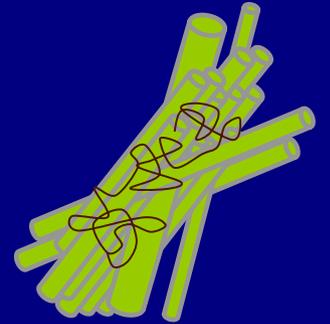
Unequal probabilities of moving in different directions  
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→ **Diffusion shape tells of structure presence and spatial orientation**

# Local Structure via Diffusion MRI

(In brief)

1) Random motion of molecules affected by local structures

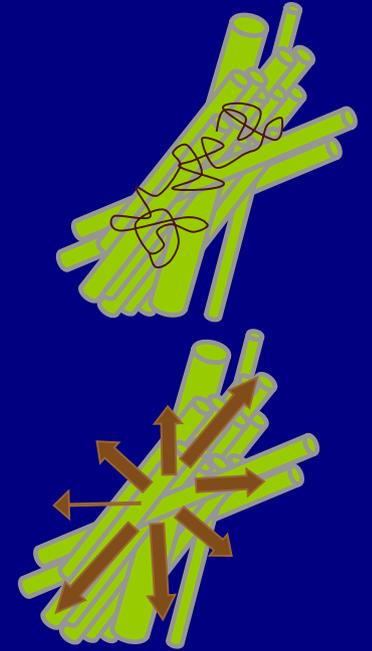


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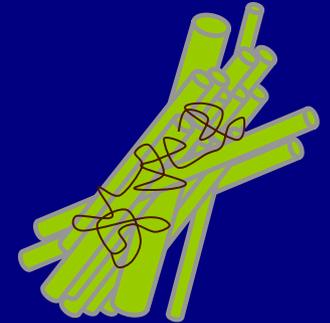
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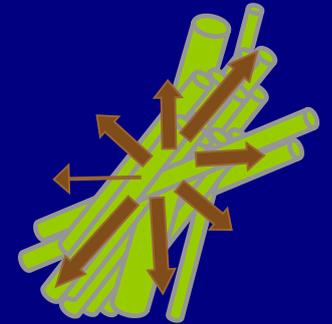
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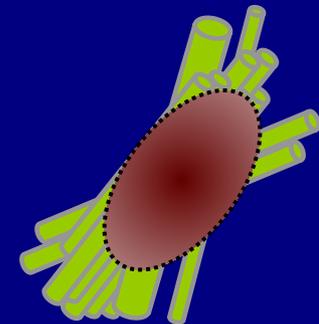
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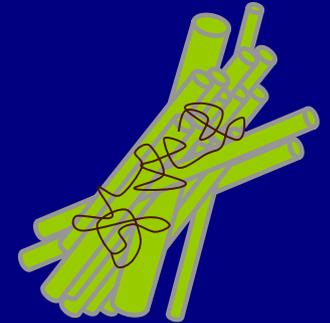
DTI (Diffusion Tensor Imaging)



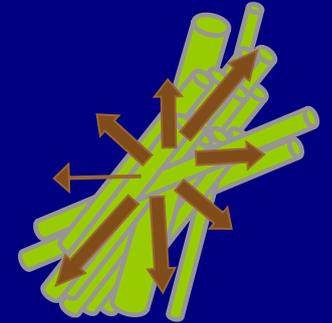
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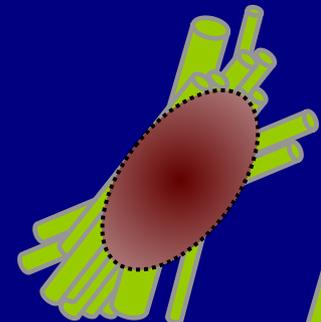
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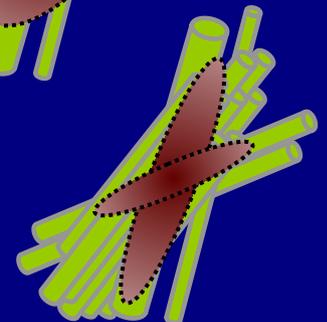
DTI (Diffusion Tensor Imaging)



+  $\geq 1$  direction:

HARDI (High Angular Resolution Diffusion Imaging)

Qball, DSI, ODFs, ball-and-stick, multi-tensor, CSD, ...



# Diffusion in MRI

Mathematical properties  
of the matrix/tensor:

$$\mathbf{D} = \begin{pmatrix} D_{11} & D_{12} & D_{13} \\ D_{21} & D_{22} & D_{23} \\ D_{31} & D_{32} & D_{33} \end{pmatrix}$$

Having: 3 eigenvectors:  $\mathbf{e}_i$   
3 eigenvalues:  $\lambda_i$

- Real-valued
- Positive definite ( $\mathbf{r}^T \mathbf{D} \mathbf{r} > 0$ )  
 $\mathbf{D} \mathbf{e}_i = \lambda_i \mathbf{e}_i, \quad \lambda_i > 0$
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Geometrically, this describes  
an ellipsoid surface:

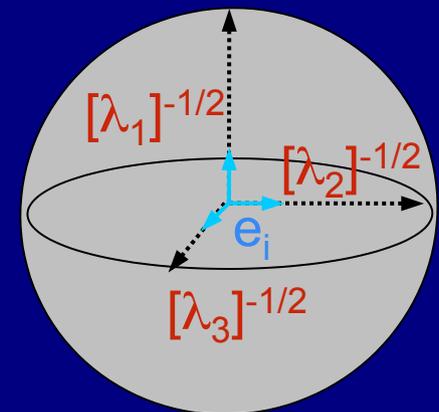
$$C = D_{11}x^2 + D_{22}y^2 + D_{33}z^2 + 2(D_{12}xy + D_{13}xz + D_{23}yz)$$

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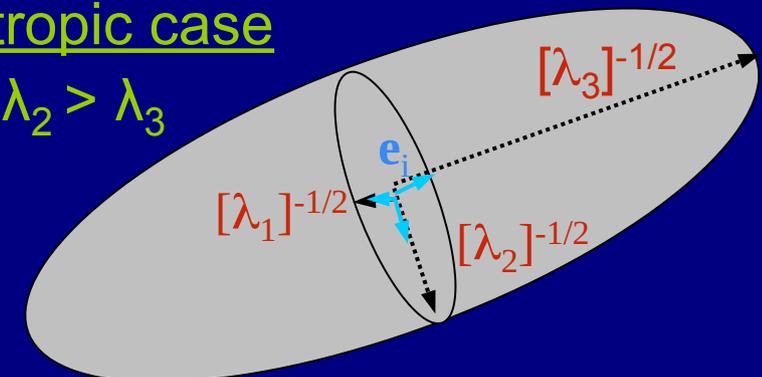
isotropic case

$$\lambda_1 = \lambda_2 = \lambda_3$$



anisotropic case

$$\lambda \lambda_1 > \lambda_2 > \lambda_3$$



# DTI: ellipsoids

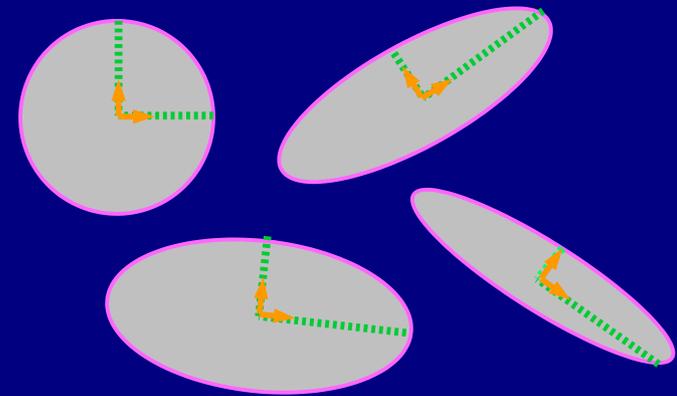
Important mathematical properties of the diffusion tensor:

+ Help to picture diffusion model:

tensor  $\mathbf{D}$   $\rightarrow$  **ellipsoid surface**

eigenvectors  $\mathbf{e}_i$   $\rightarrow$  orientation in space

eigenvalues  $\lambda_i$   $\rightarrow$  'pointiness' + 'size'



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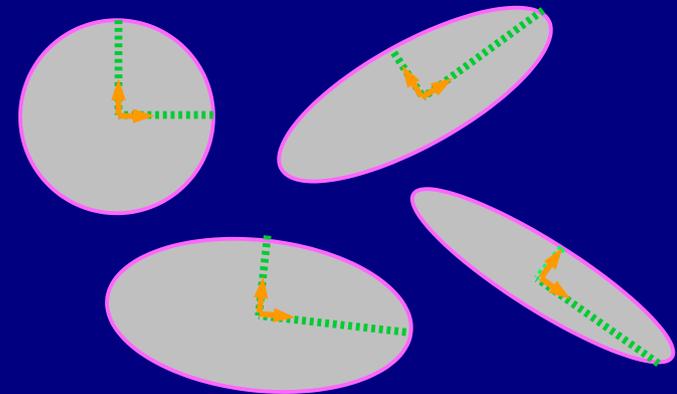
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+ Determine the minimum number of

DWIs measures needed (6 + baseline)

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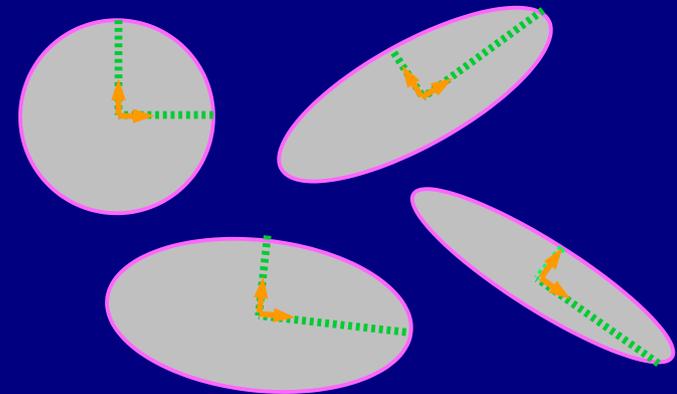
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+ Determine much of the processing and

noise minimization steps

$D_{11}$	$D_{12}$	$D_{13}$
$D_{21}$	$D_{22}$	$D_{23}$
$D_{31}$	$D_{32}$	$D_{33}$

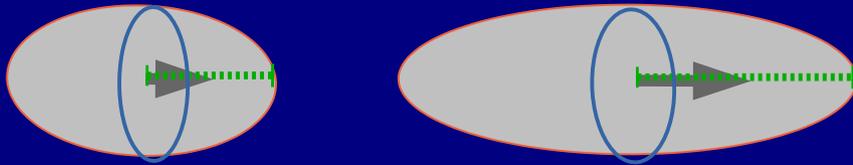
# “Big 5” DTI ellipsoid parameters

## Main quantities of diffusion (motion) surface

---

first eigenvalue,  $L1$

(=  $\lambda_1$ , parallel/axial diffusivity,  $AD$ )



$L1_1$

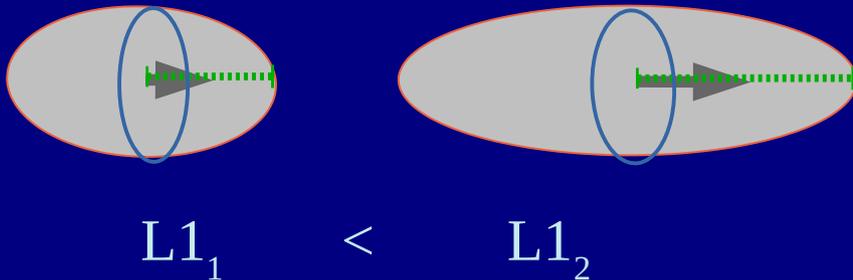
<

$L1_2$

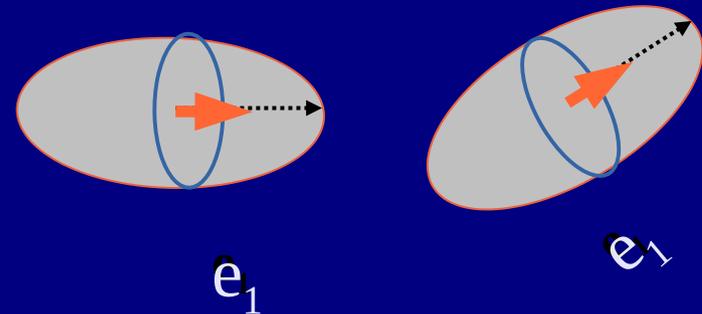
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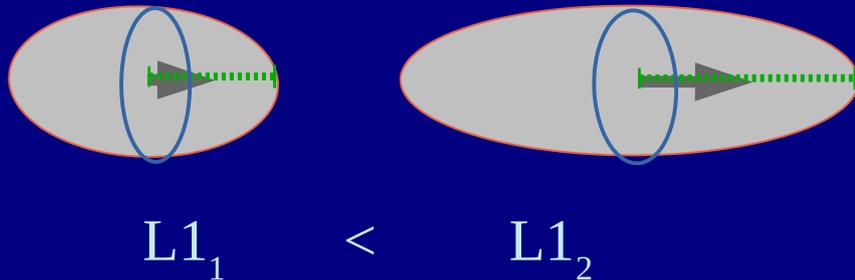
first eigenvector,  $e_1$   
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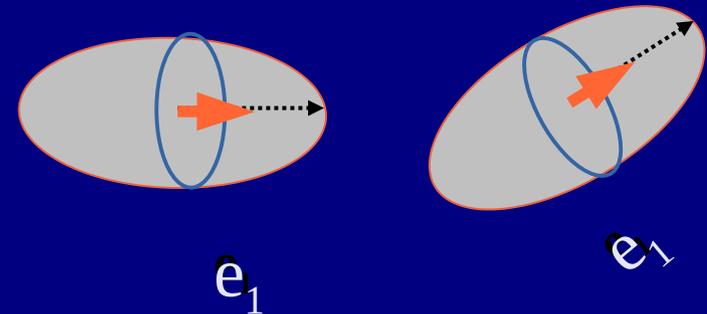
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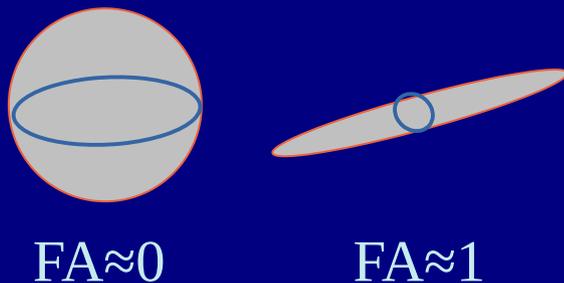
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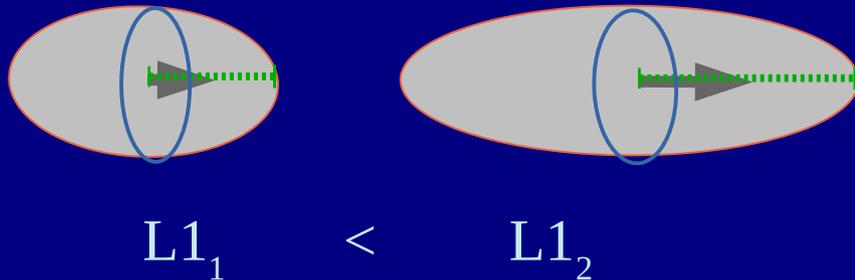
Fractional anisotropy,  $FA$   
(stdev of eigenvalues)



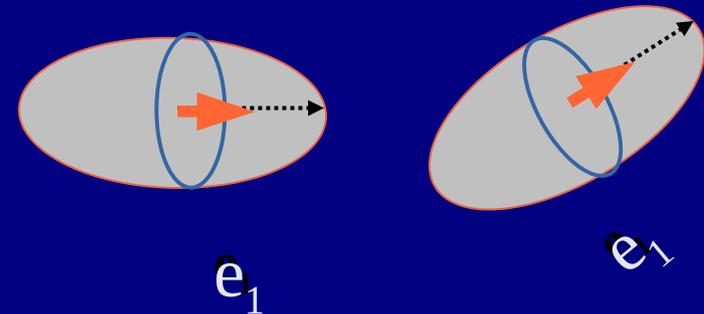
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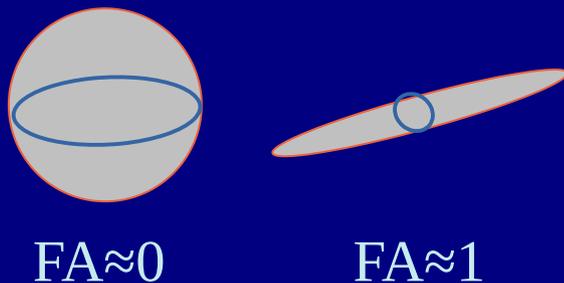
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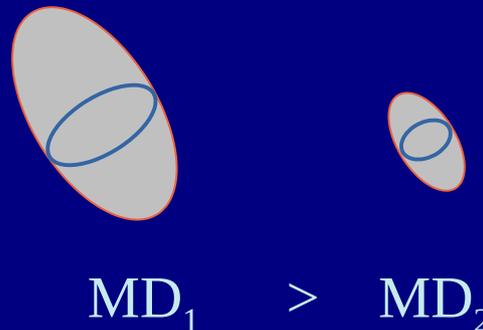
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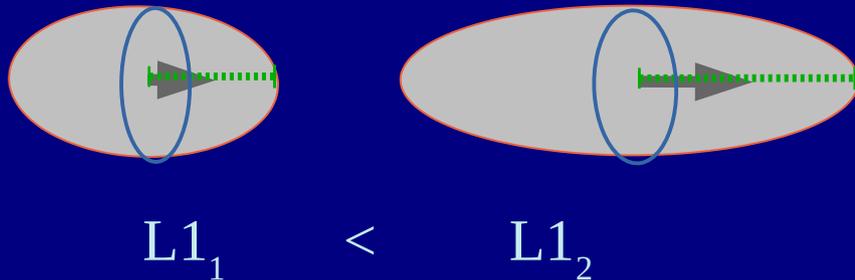
Mean diffusivity,  $MD$   
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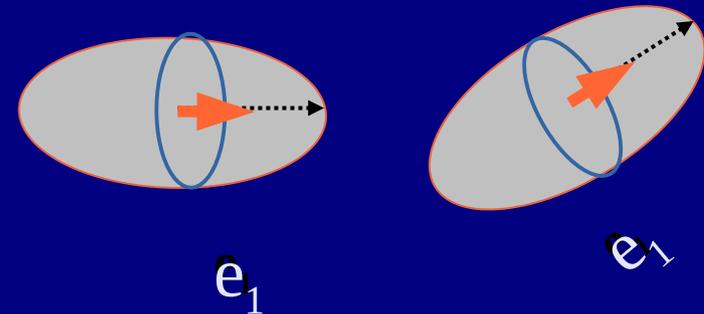
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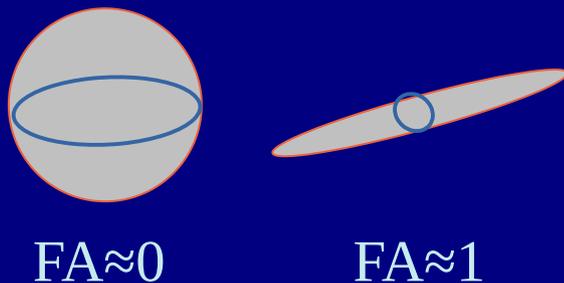
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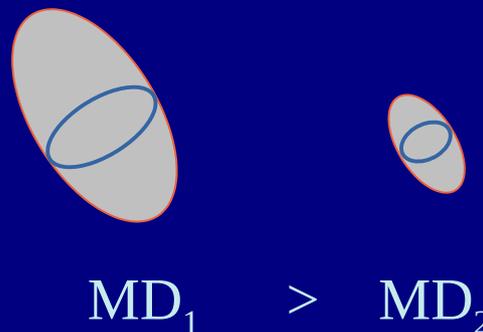
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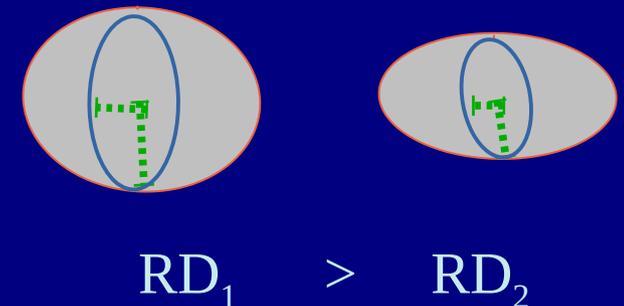
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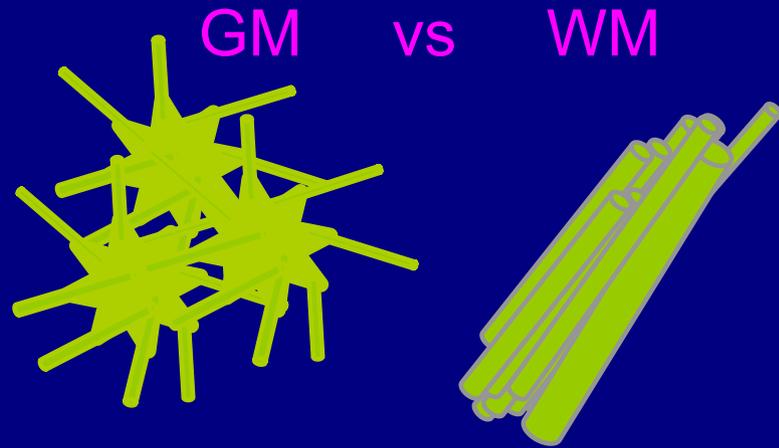
Mean diffusivity,  $MD$   
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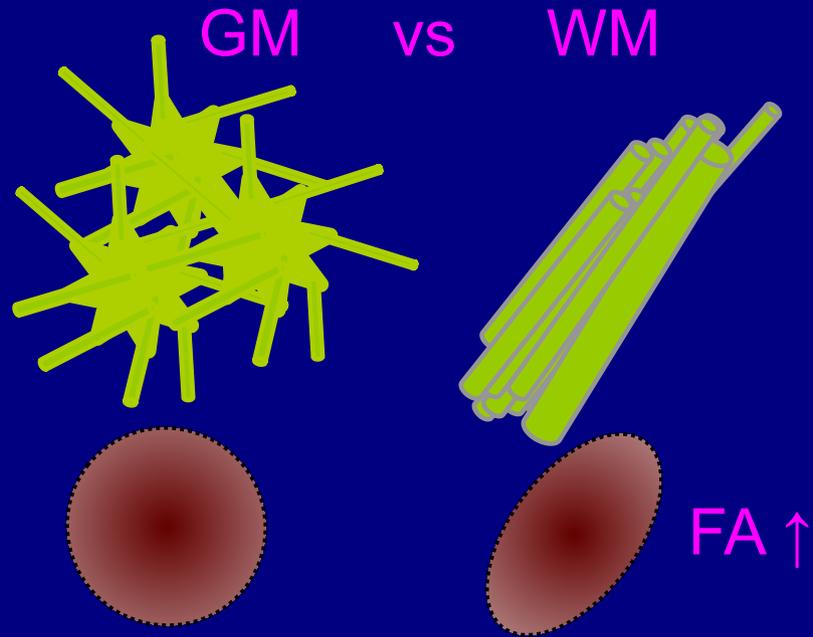
Radial diffusivity,  $RD$   
(=  $(\lambda_2 + \lambda_3)/2$ )



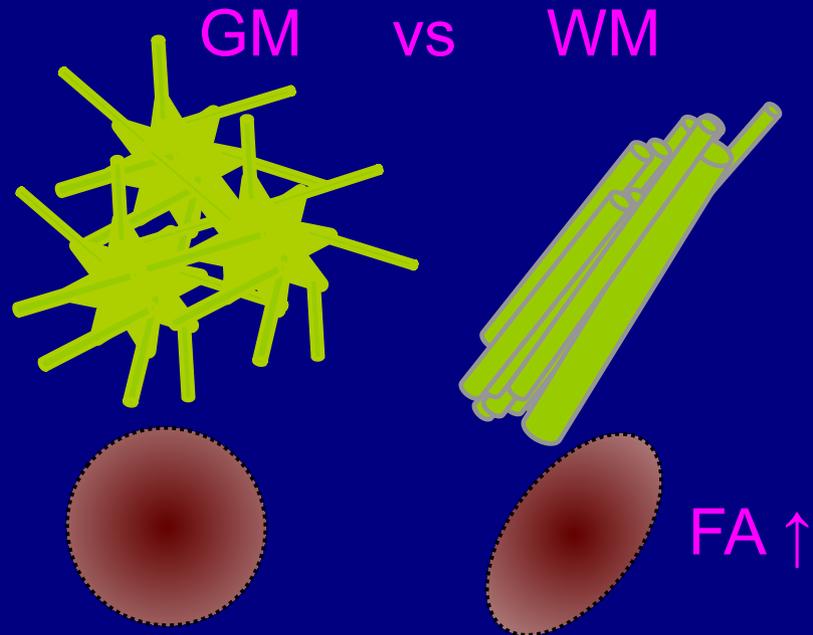
# Cartoon examples: white matter $\leftrightarrow$ FA



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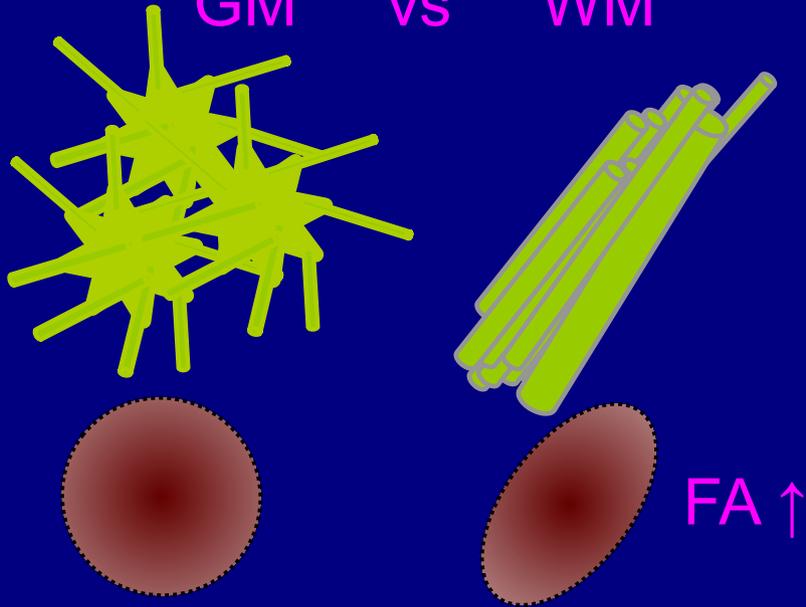


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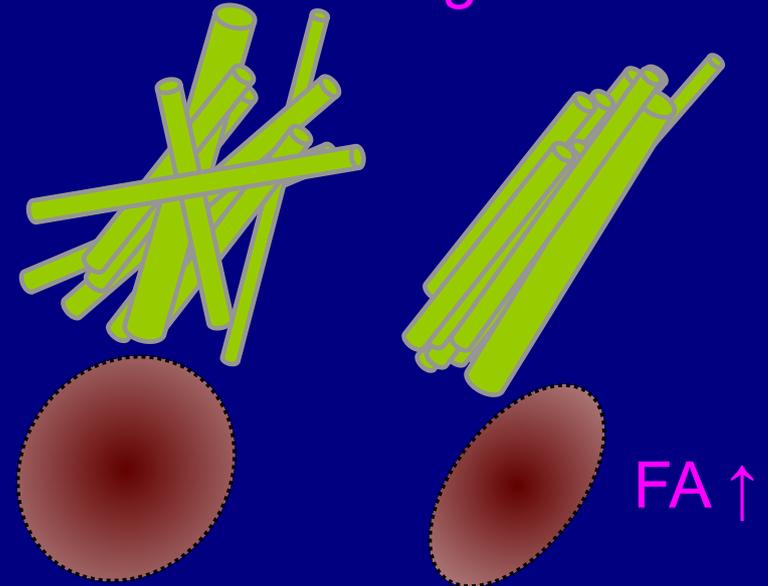


# Cartoon examples: white matter $\leftrightarrow$ FA

GM vs WM

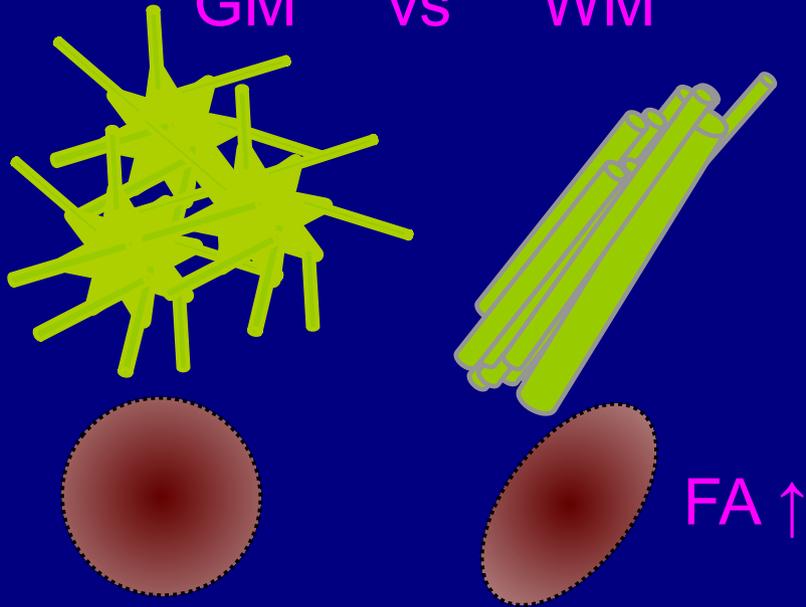


WM bundle organization

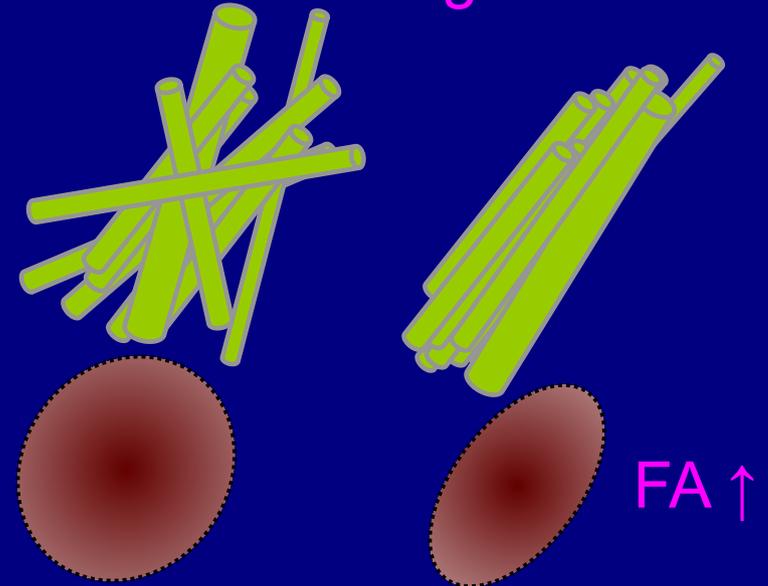


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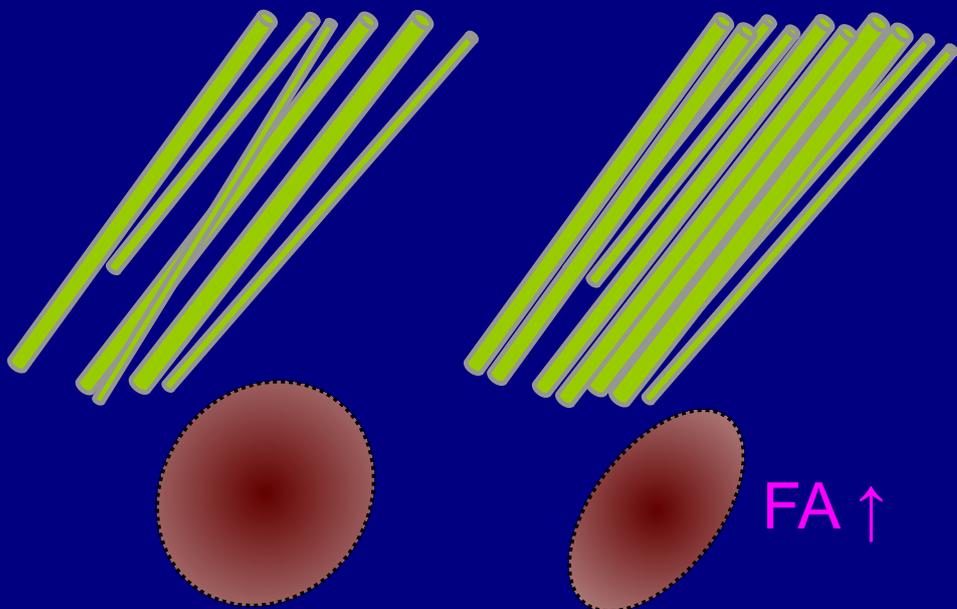
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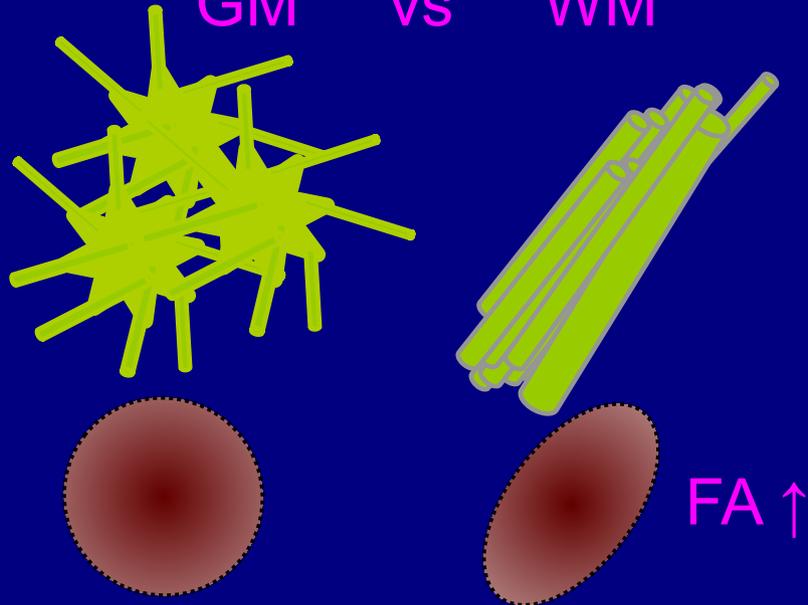


WM bundle density

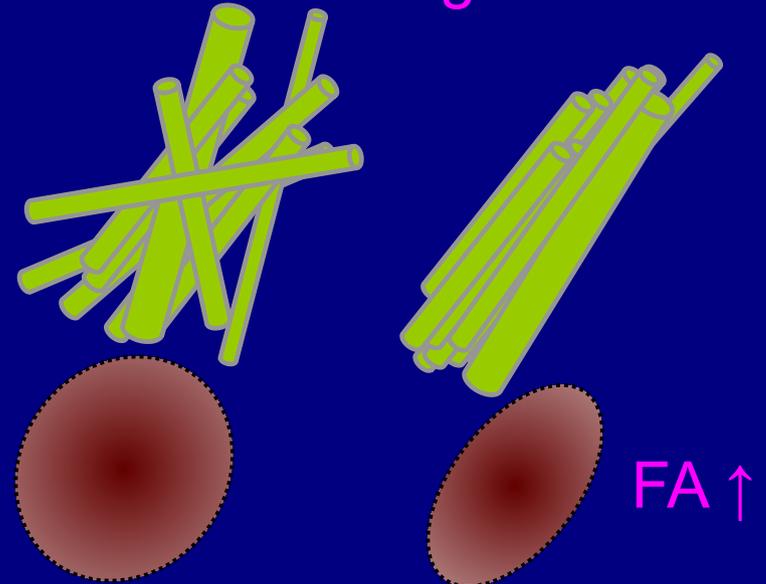


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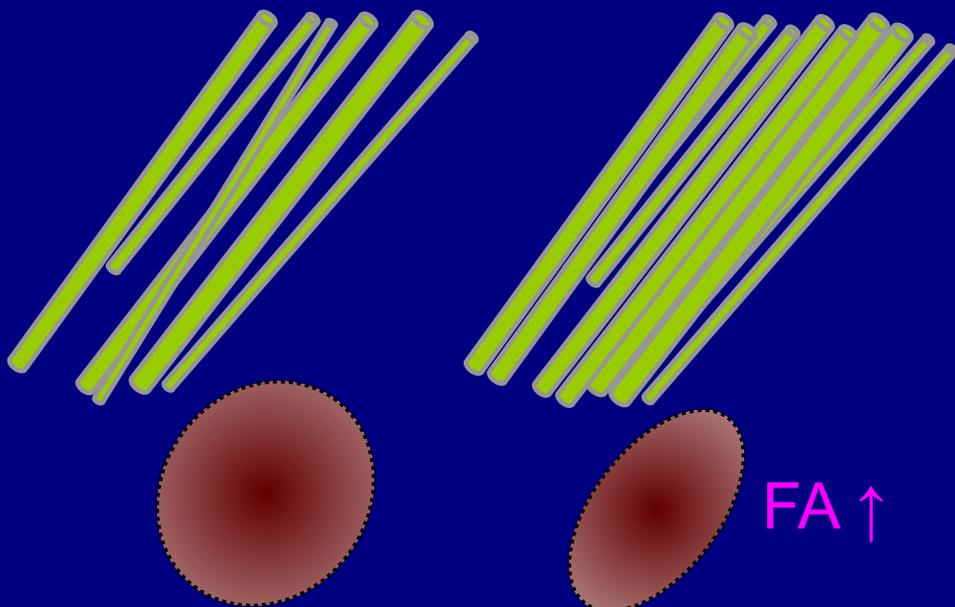
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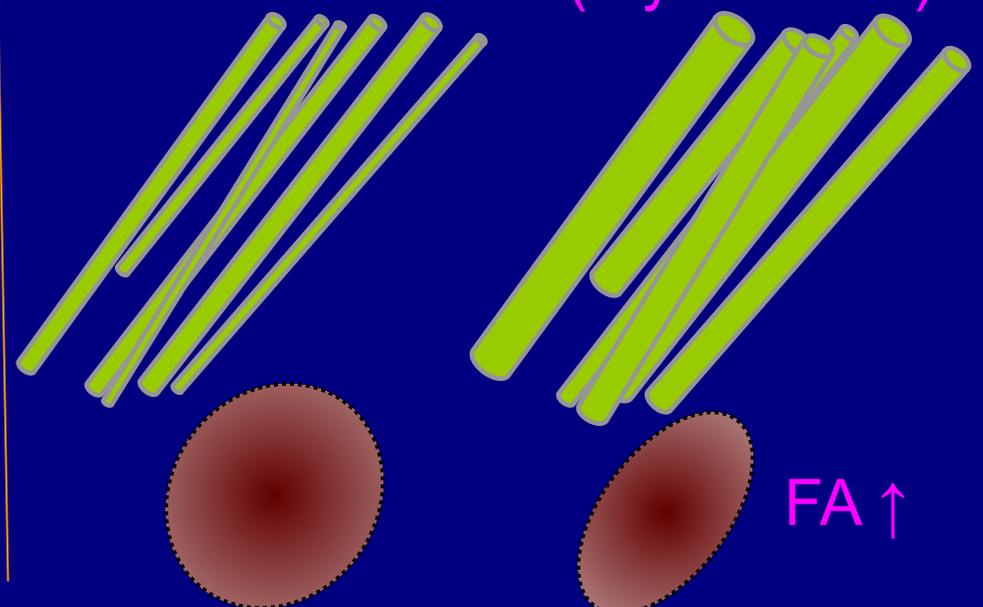
WM bundle organization



WM bundle density



WM maturation (myelination)



# Interpreting DTI parameters

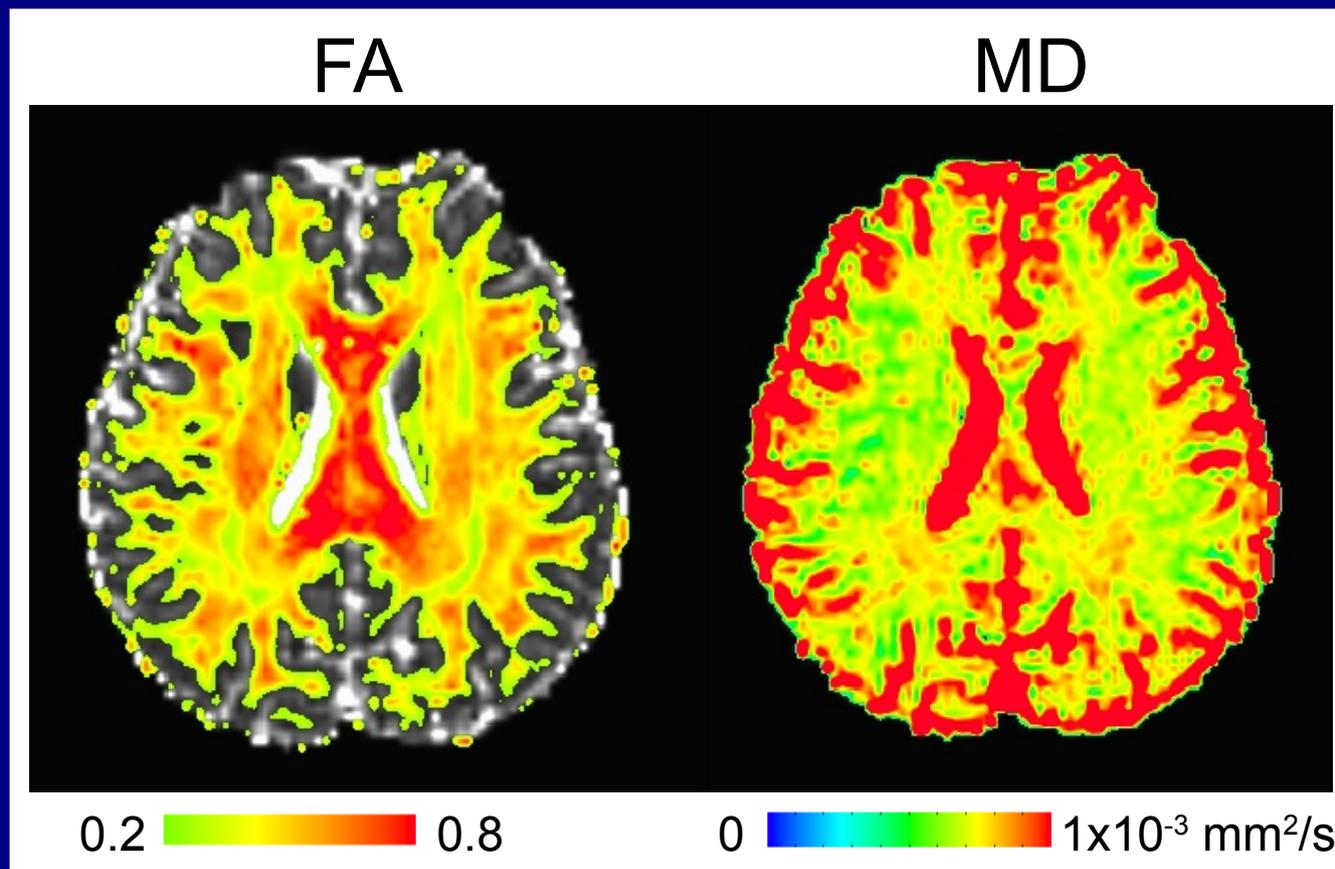
## General literature:

**FA**: measure of fiber bundle coherence and myelination

- in adults,  $FA > 0.2$  is proxy for WM

**MD, L1, RD**: local density of structure

**$e_1$** : orientation of major bundles



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**$e_1$** : orientation of major bundles

## Cautionary notes:

- Degeneracies of structural interpretations
- Changes in myelination may have small effects on FA
- WM bundle diameter  $\ll$  voxel size
  - don't know location/multiplicity of underlying structures
- More to diffusion than structure-- e.g., fluid properties
- Noise, distortions, etc. in measures

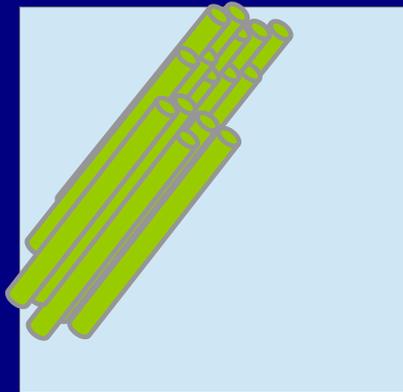
*Acquiring DTI data:*  
diffusion weighted gradients in MRI

# Diffusion weighted imaging

For a given voxel, observe relative diffusion along a given 3D spatial orientation (gradient)

DW gradient

$$\mathbf{g}_i = (g_x, g_y, g_z)$$

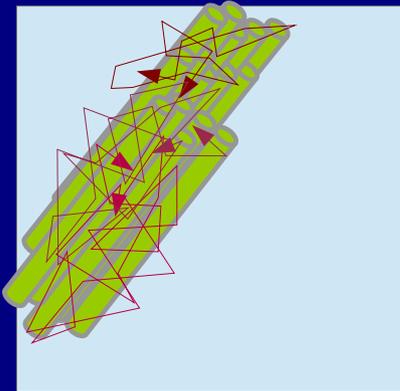


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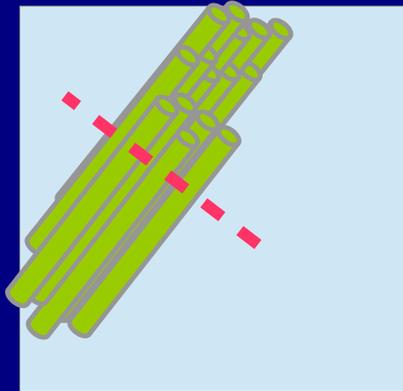
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MR signal is attenuated by diffusion throughout the voxel in that direction:

$$S_i = S_0 e^{-b \mathbf{g}_i^T \mathbf{D} \mathbf{g}_i}$$

→ ellipsoid equation of diffusion surface:  
 $\mathbf{C} = \mathbf{r}^T \mathbf{D}^{-1} \mathbf{r}$ .



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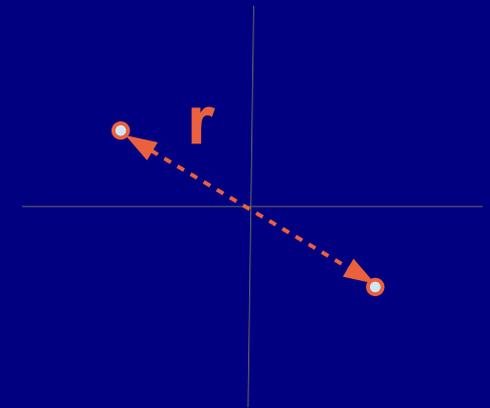
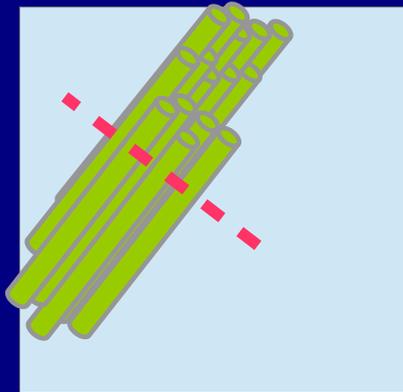
$$\mathbf{g}_i = (g_x, g_y, g_z)$$

diffusion

motion

ellipsoid:

$$C_2 = \mathbf{r}^T \mathbf{D}^{-1} \mathbf{r}.$$



# Diffusion weighted imaging

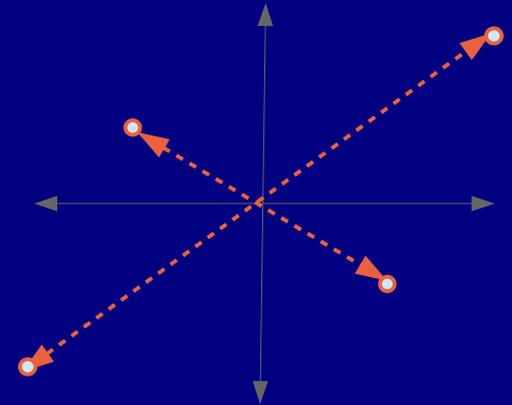
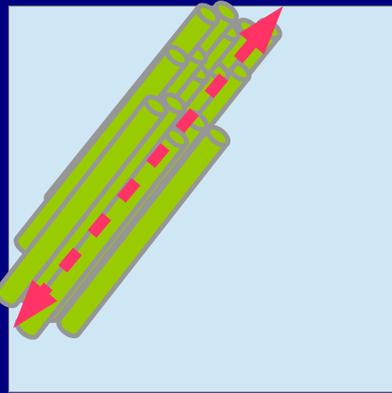
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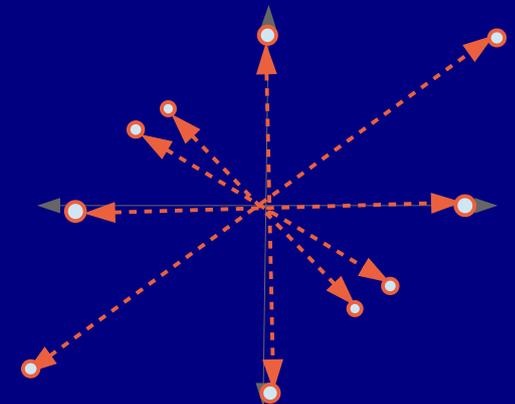
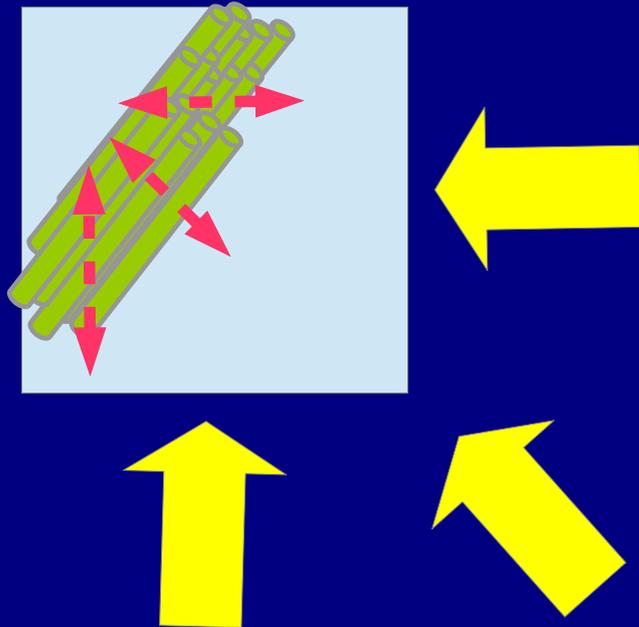
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ellipsoid:

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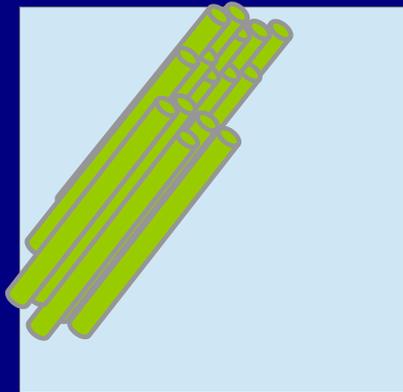


# Diffusion weighted imaging

For a given voxel, observe relative diffusion along a given 3D spatial orientation (gradient)

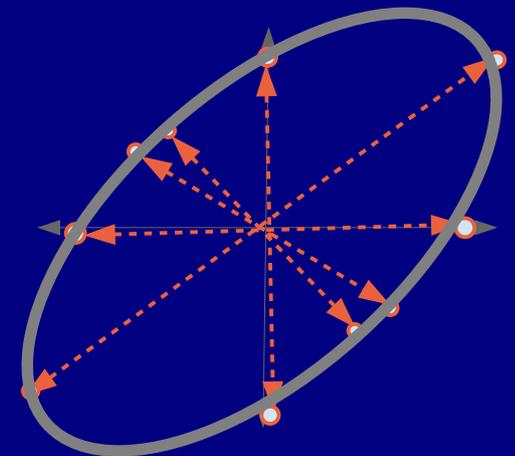
DW gradient

$$\mathbf{g}_i = (g_x, g_y, g_z)$$



diffusion  
motion  
ellipsoid:

$$C_2 = \mathbf{r}^T \mathbf{D}^{-1} \mathbf{r}.$$

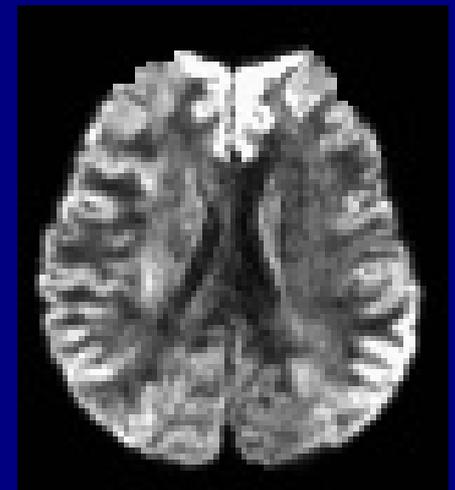
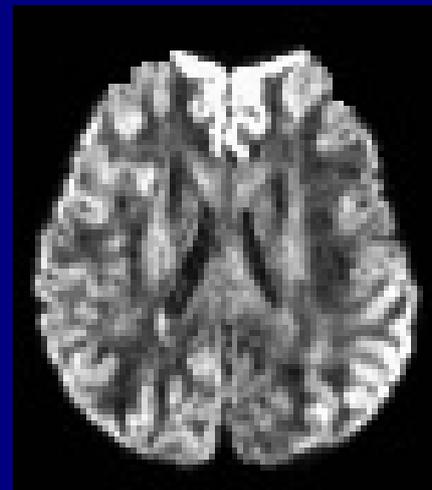
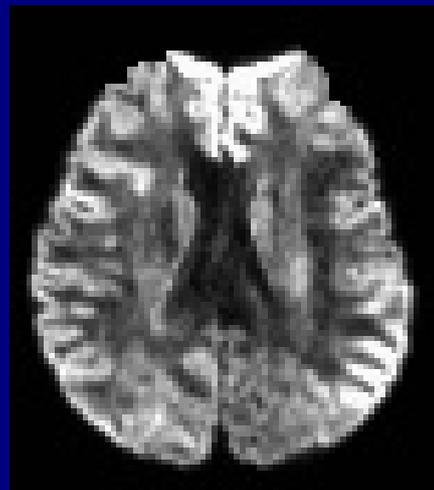
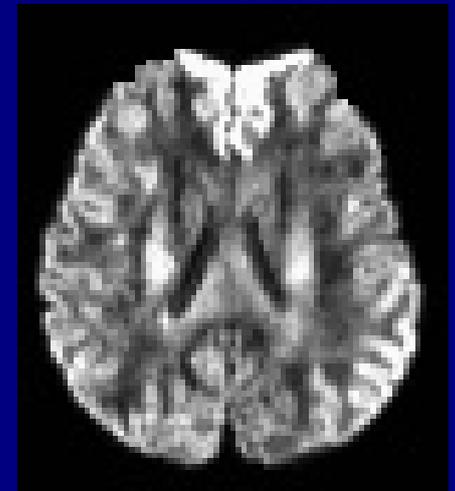
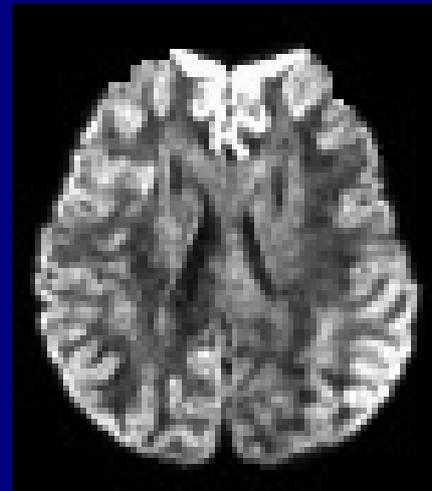
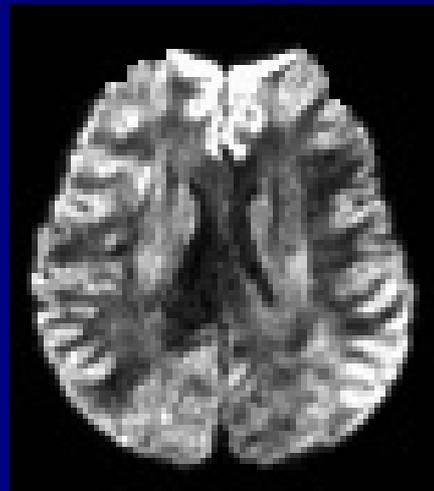
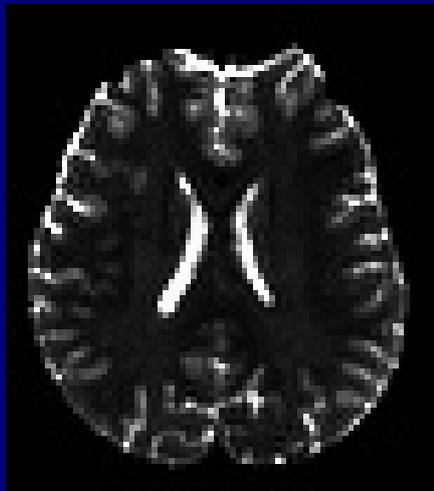


Individual points  $\rightarrow$  Fit ellipsoid surface  
Individual signals  $\rightarrow$  Solve for  $\mathbf{D}$

# Sidenote: what DWIs look like

Unweighted  
reference  
 $b=0$  s/mm<sup>2</sup>

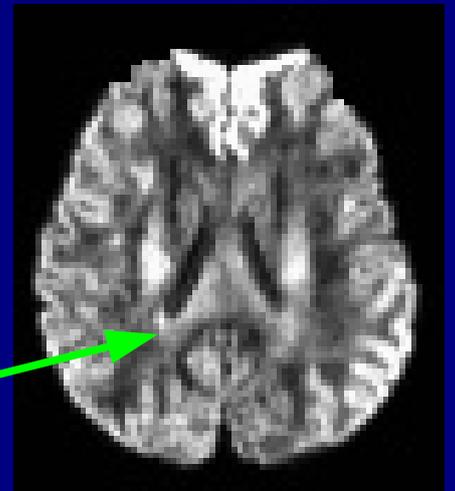
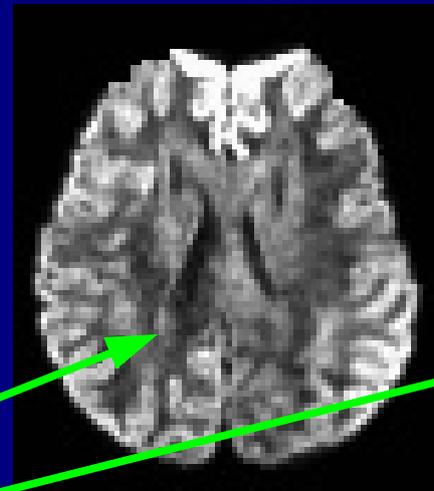
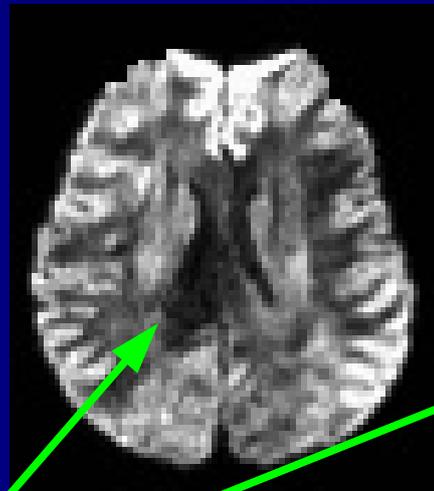
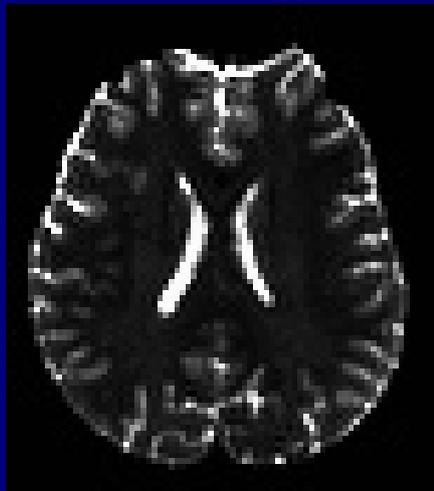
Diffusion weighted images  
(example:  $b=1000$  s/mm<sup>2</sup>)



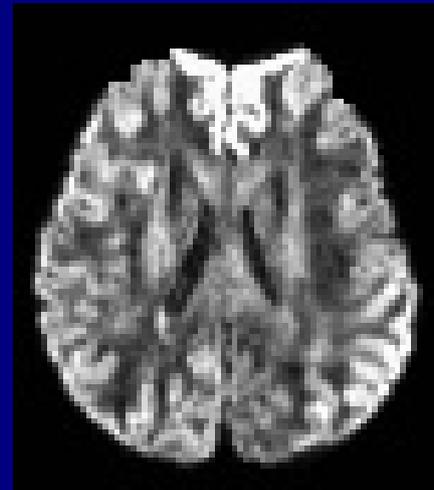
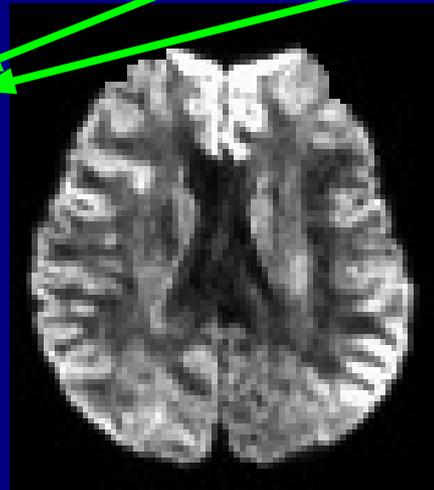
# Sidenote: what DWIs look like

Unweighted  
reference  
 $b=0 \text{ s/mm}^2$

Diffusion weighted images  
(example:  $b=1000 \text{ s/mm}^2$ )



(Each DWI has a  
different brightness  
pattern: viewing  
structures from  
different angles.)



# Noise in DW signals

MRI signals have additive noise

$$S_i = S_0 e^{-b \mathbf{g}_i^T \mathbf{D} \mathbf{g}_i} + \varepsilon,$$

where  $\varepsilon$  is (Rician) noise.

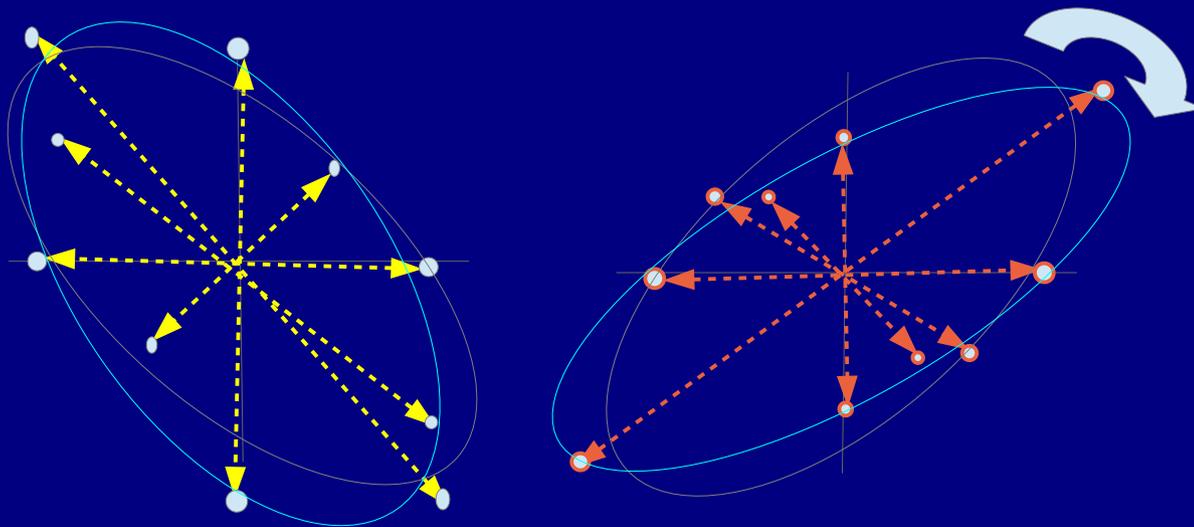
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→ Leads to errors in surface fit, equivalent to *rotations* and *rescalings* of ellipsoids:



'Un-noisy' vs perturbed/noisy fit

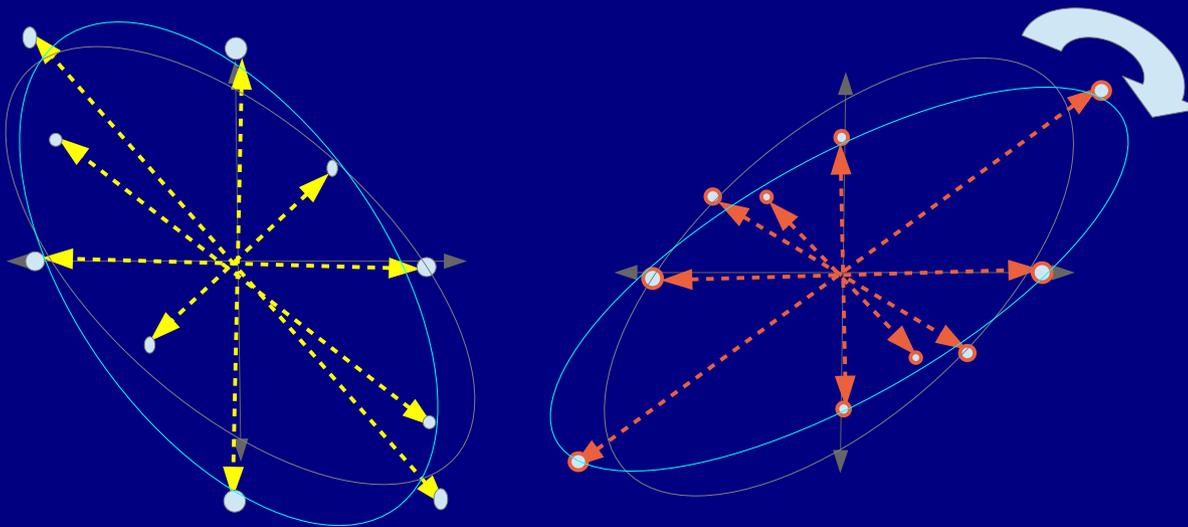
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'Un-noisy' vs perturbed/noisy fit

Leads to standard:  
+ 30 DWs (~12 clinical)  
+ repetitions of  $b=0$   
+ DW  $b$  chosen by:  
 $MD * b \approx 0.84$   
+ nonlinear tensor fitting

# Distortions in DWI volumes

There are also **serious** sources of distortion when acquiring DWIs:

- + Subject motion

  - due to movement during/between volume acq. -> signal loss/overlap

- + Eddy current distortion

  - due to rapid switching of gradients -> nonlinear/geometric distortions

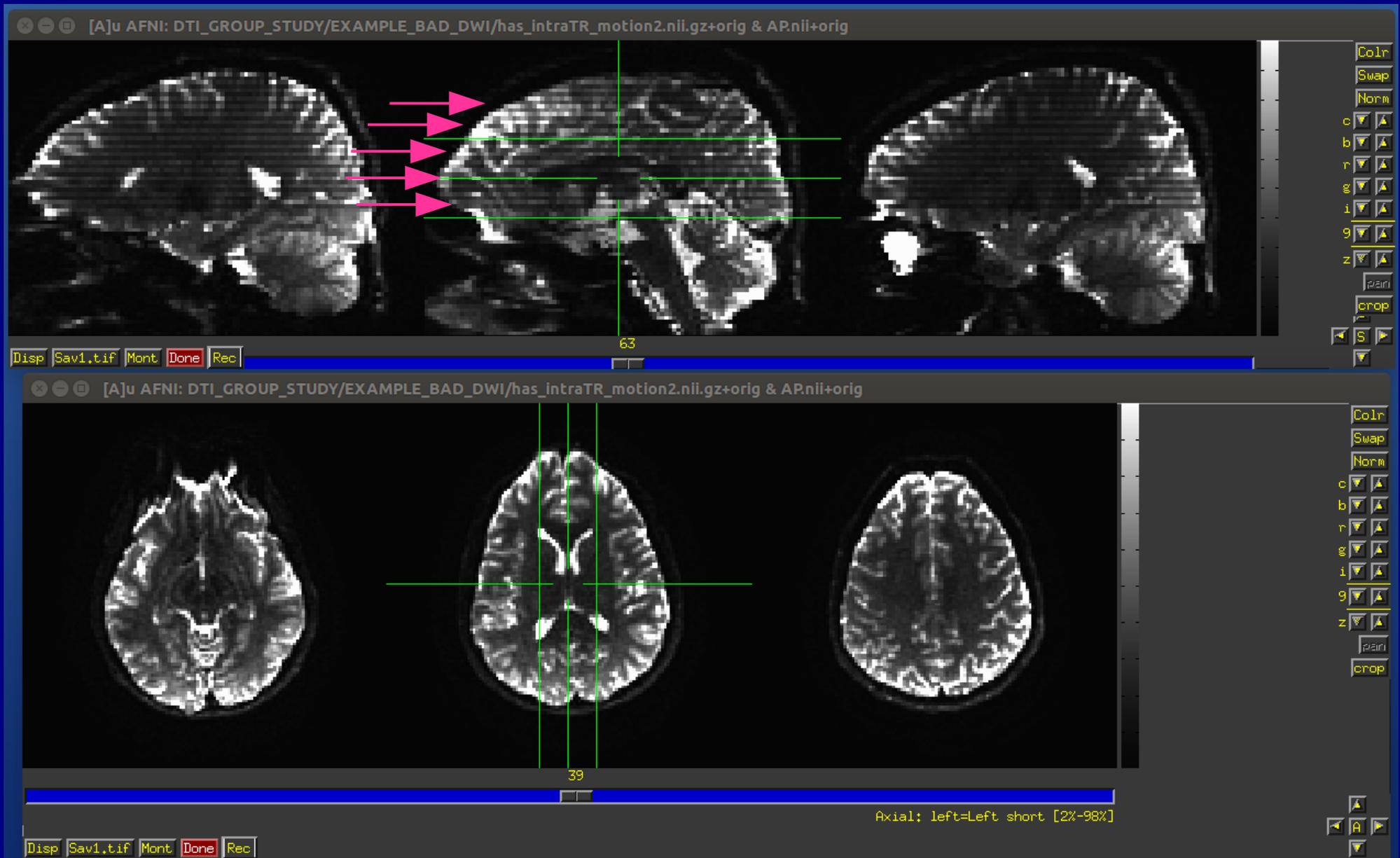
- + EPI distortion

  - due to B0 inhomogeneity -> geometric distortions along phase encoding dir, signal pileup or attenuation

---> And effects combine! Need careful acquisition (sometimes perhaps even **reacquisitions**) and post-processing.

# Distortions in DWI volumes

From subj motion: interleaved brightness distortions





# SUMMARY

- + Diffusion-based MRI uses application of magnetic field gradients to probe the relative diffusivity of molecules along different directions.
- + DTI combines that information into a simple shape family, spheroids, to summarize the diffusivity.
- + From the DT, several useful properties are described in terms of scalar (e.g., FA, MD, L1) and vector (e.g., V1) parameters.
- + Many “standard” interpretations of DTI parameters exist (i.e., higher FA = “better” WM), but we must be cautious.
- + Distortions and noise affect all DTI estimates, and we must consider the consequences of these in all analyses.

